2007 International Symposium on Nonlinear Theory and its Applications
NOLTA'07, Vancouver, Canada, September 16-19, 2007

Chaos in a Switched Dynamical System: Scicos as a Modeler and Simulator

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Abstract– In this paper, we explore some modeling capabilities of Scicos. Our aim is to generate chaos from a simple hybrid dynamical system. We give the chaotic dynamics of the current-programmed boost converter in open-loop circuit as a case study. By considering two bifurcation parameters the current reference and the voltage input, we observe that the obtained Scicos simulations show that the boost converter is prone to subharmonic behavior and chaos. We also present the corresponding bifurcation diagrams. We use Scicos because it provides a much clearer understanding of the way the model will behave during simulation reducing considerably the risk of ambiguity.

1. Introduction

Hybrid dynamical systems (HDS) have attracted considerable attention in recent years. HDS arise from the interaction between continuous variable systems (i.e., systems that can be described by a difference or differential equation) and discrete event systems (i.e., systems where the state transitions are initiated by events that occur at discrete time instants).

A simple example is a thermostat in a room. The temperature in the room is a continuous variable, and the state of the thermostat ("on" or "off") is discrete; the continuous and discrete parts cannot be described independently since clearly there is interaction between the two. Some other examples include walking robots, nonlinear electronic circuits, biological cell growth and division, chemical plant controlled with valves and pumps, aircraft with a switching controller.

Switched piecewise linear systems are an important class of hybrid systems that are simple and can have very rich and typical nonlinear dynamics such as bifurcations and chaos.

As example, DC–DC switching converters are switched piecewise linear systems [1]. The three basic power electronic converters buck, boost and buck-boost are variable structure systems that are highly nonlinear. The kind of piecewise model may present nonlinear phenomena such as bifurcations and chaos.

The study of nonlinear dynamics of DC-DC converters started in 1984 by Brockett's and Wood's research [2]. Since then, chaos and nonlinear phenomena in power electronic circuits have stolen the spotlight and have attracted the attention of different research groups. Different nonlinear phenomena were investigated such as flip bifurcation or period doubling and its related route to chaos [3-11] or quasiperiodicity route to chaos [13, 14], as well as border collision bifurcation [11, 12, 15].

There are many modeling techniques, programming languages, and design toolsets for HDS. To model and simulate our HDS, we use Scicos (Scilab Connected Object Simulator) which is a Scilab package for modeling and simulation of dynamical systems including both continuous and discrete time subsystems [16, 17]. Scilab (Scientific Laboratory) is a scientific software package for numerical computations that provides a powerful open computing environment for engineering and scientific applications [18]. It has been developed at INRIA and ENPC and is freely available for download at "http://www.scilab.org". One of the other advantages of Scicos is that it provides a much clearer understanding of the way the model will behave during simulation reducing considerably the risk of ambiguity.

This paper aims to study and analyze some dynamic phenomena that can occur in the current-mode controlled boost converter. We also show from Scicos simulations that variation of the current reference or the voltage input can lead to interesting route to chaos.

In section 2, the general equation of a hybrid dynamical system is briefly recalled. In Section 3, we explain the operation of the current-mode controlled boost converter. Then, we introduce the state equations of the circuit in question. In section 4, we comment on the obtained Scicos simulations. We end by some concluding remarks.

2. Hybrid Dynamical System

The evolution of an autonomous hybrid dynamical system can be described by:

$$\frac{dx}{dt}(t) = f(x(t),q(t)), \quad x(t_0) = x_0,$$
$$q(t) = e(x(t),q(t-)), \quad q(t_0) = i_0,$$

where $x(t) \in \Re^n$ is the continuous state vector, $q(t) \in Q=\{1,\dots n_Q\}$ denotes the discrete state and q(t-) is the previous discrete state. The state space is $H=\Re^n \times Q$ and the initial state is supposed belonging to the set of initial conditions $(x_0,i_0)\in H_0 \subseteq H$. The function $e:\Re^n \times Q \rightarrow Q$ describes the change of the discrete state. The change from one distinct discrete state to another is called a transition or a switch. A transition between two states *i* and *j* occurs if *x(.)* reaches the switch set $S_{i,j}$

$$S_{i,j} = \{x : e(x,i) = j\}.$$

Among important classes of hybrid systems, we find piecewise linear systems that are described by:

$$\frac{dx}{dt}(t) = f_q(x(t)) = A(q)x(t) + B(q)$$

where $A(q) \in \Re^{n\times m}$ and $B(q) \in \Re^n$ are matrices depending on q.

3. Current-Mode Controlled Boost Converter in Open Loop

3.1. Operation of Current-Mode Controlled Boost Converter in Open Loop

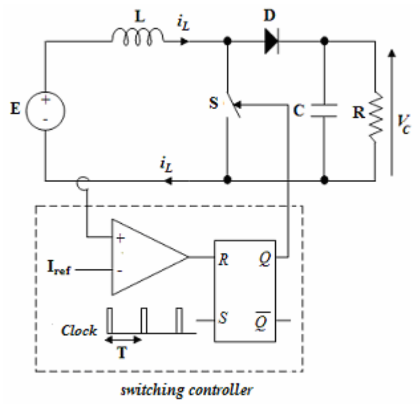

Fig. 1. Current-mode controlled boost converter

A current-mode controlled boost converter in open loop consists of two parts: a converter and a switching controller. The basic circuit is given in Fig. 1.

The converter is a second-order circuit comprising an inductor, a capacitor, a diode, a switch and a load resistance connected in parallel with the capacitor.

The general circuit operation is driven by the switching controller. It compares the inductor current i_L with the reference current I_{ref} and generates the on/off driving signal for the switch S. When S is on, the current builds up in the inductor. When the inductor current i_L reaches a reference value, the switch opens and the inductor current flows through the load and the diode. The switch is again closed at the arrival of the next clock pulse from a free running clock of period T.

3.2. State Equations

When operating in continuous conduction mode (CCM), two switch states can be identified:

- Switch on and diode off
- Switch off and diode on

Whether the switch is on or off, the boost converter can always be described as a second order linear system, whose states are the voltage v_C across the capacitor, and the current i_L along the inductor.

The general equation that models operation of the boost converter takes the form:

$$\frac{dx}{dt}(t) = f_q(x(t)) = A(q)x(t) + B(q) \quad with \quad q \in Q = \{1,2\}$$

For *q=1* and *q=2,* we obtain the following two systems of differential equations:

$$S_{on} : \frac{dx}{dt} = f_1(x) = A_1 x + B_1$$

$$S_{off} : \frac{dx}{dt} = f_2(x) = A_2 x + B_2$$

where $x = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$ is the vector of the state variables. The A's and B's matrices and vectors are given by:

$$A_1 = \begin{bmatrix} \frac{-1}{RC} & 0 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{E}{L} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{E}{L} \end{bmatrix}$$

The switching borders of each sub-system S_{on} and S_{off} are given by:

$$B_1 = \{(x,t) \in R^2 \times R : \beta_{on,off}(x) = i_L - I_{ref} = 0\}$$

$$B_2 = \{(x,t) \in R^2 \times R : \beta_{off,on}(x,t) = t - nT = 0\}$$

where $\beta_{on,off}$ and $\beta_{off,on}$ are the switching conditions. B_1 is a fixed border ($\beta_{on,off}(x)=0$ is a nonautonomous function) whereas B_1 is a moving border ($\beta_{off,on}(x)=0$ is an autonomous function).

The boost converter in CCM switches between two systems S_{on} and S_{off} if the borders B_1 and B_2 are reached. Fig. 2 shows the transition diagram.

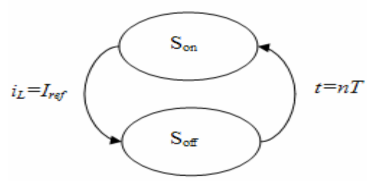

Fig. 2. State diagram of the boost converter

4. Simulation Results and Comments

We choose the parameter values: L= 1.5mH, T= 100µs, R= 40Ω and C= 5µF. We consider two parameters of bifurcation: the current reference I_{ref} and the input voltage

E. By varying I_{ref} or E, the circuit changes its qualitative behavior from a stable periodic system to another system that exhibits chaos.

4.1. Transition to Chaos by Varying the Current Reference I_{ref}

In this paragraph we analyze the behavior of the boost converter by varying the current reference I_{ref} , whereas the input voltage E is chosen as E=10V.

At first, using Scicos we draw the one-parameter bifurcation diagram given in Fig. 3 where the reference current I_{ref} is the bifurcation parameter and the sampled i_L is the variable. By increasing I_{ref}, the displayed diagram shows a period doubling bifurcation with occurrence of border collision bifurcation at the critical point I_{ref}=1.23A after period-2 solution. Then the system jumps to period-4 solution and pursues period doubling route to chaos.

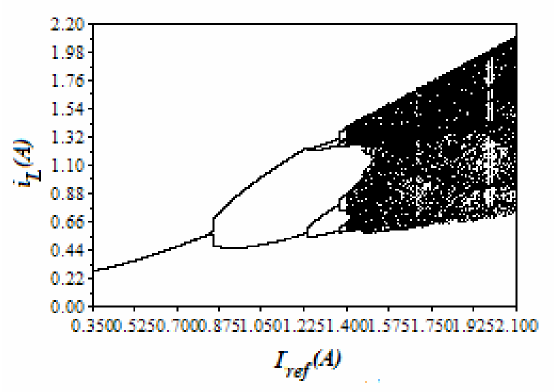

Fig.3. Bifurcation diagram with fixed E=10V

By increasing I_{ref}, we give for different current reference I_{ref} the inductor current wave form i_L and its corresponding phase plan v_C-i_L.

By choosing I_{ref} = 0.7A, Fig. 5 shows the fundamental periodic operation. Fig. 5(a) displays the inductor current wave form, Fig. 5(b) gives the corresponding phase plan, and these figures prove the stable and periodic nature of the circuit. This periodic regime is possible just for small I_{ref} values.

For I_{ref} = 1A and I_{ref} = 1.3A, subharmonic operation has been found. The chaotic operation is given in Fig. 6: (a) indicates a chaotic signal with infinite order and (b) the phase plan v_C-i_L corresponds to a chaotic attractor.

4.2. Transition to Chaos by varying the input voltage E

This part studies the dynamic behavior of the boost converter. The current reference value I_{ref} is chosen as I_{ref}=0.8A and the input voltage E is varied.

We start our numerical study by representing the one-parameter bifurcation diagram with the choice of the input voltage E as a bifurcation parameter. Decreasing E in our corresponding Scicos model gives Fig. 4 that shows the bifurcation diagram. We observe that border collision bifurcation happens at the critical point E = 6.51V before period-2 solution. If we continue decreasing E, the circuit jumps to period-4 solution and pursues period doubling route to chaos.

Thus, we can observe many kinds of periodic regimes or a chaotic regime. This is also justified by displaying the inductor current wave form i_L and its corresponding phase plan v_C-i_L.

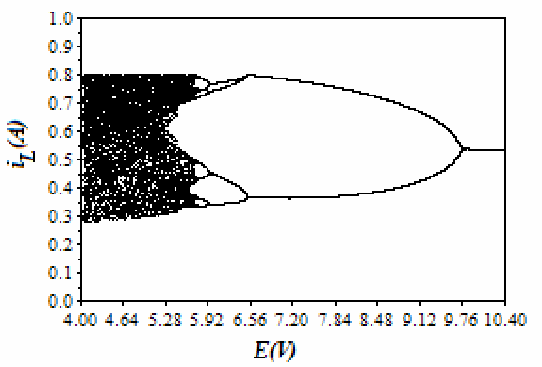

Fig. 4. Bifurcation diagram with fixed I_{ref} =0.8A

The fundamental periodic operation was obtained for E=11V. However 2-T periodic operation and 4-T periodic one were obtained respectively for E=8V and E=6.1V. While, when we fix the input voltage E at 4.4V, we observe the chaotic regime.

5. Conclusion

This article has illustrated a Scicos numerical study of the current-mode controlled boost converter that is modeled by a hybrid system. Variations of the current reference or the voltage input can lead to interesting route to chaos; the system pursues period doubling bifurcation with occurrence of border collision bifurcation at critical points. The displayed simulation results prove also that Scicos is a powerful simulator of hybrid systems.

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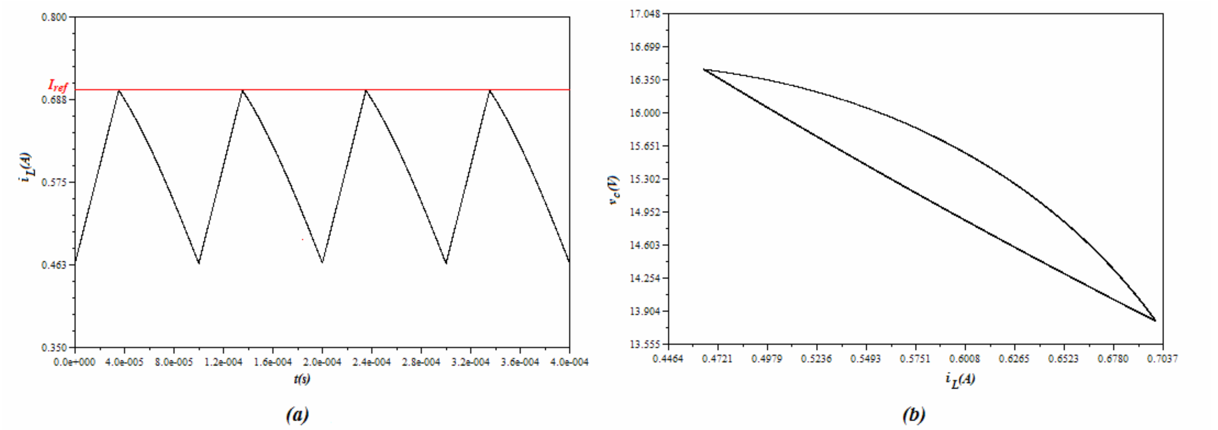

Fig. 5. Fundamental periodic operation (I_{ref} = 0.7A). (a) Time waveform of the inductor current (b) Phase plan

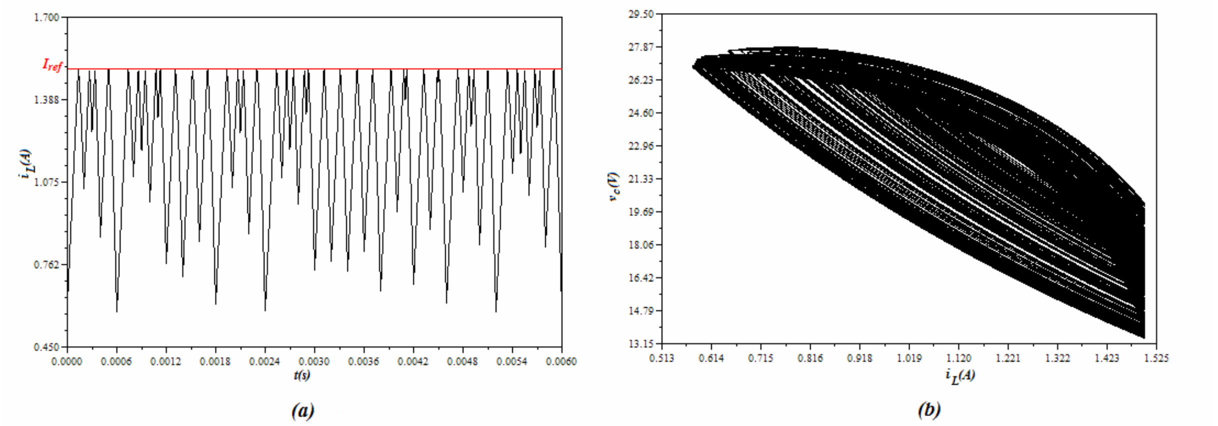

Fig. 6. Chaotic Regime (I_{ref} = 1.5A). (a) Time waveform of the inductor current (b) Phase plan